Report
to
Professor Katsuhisa Furuta

by
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Introduction

Review of literatures on Distributed Parameters System

In [CI74], the existence and the stability is studied for a boundary value problem which is described as a dynamical system in Banach space. A system of partial differential equation is interpreted as a dynamical system defined on an appropriate phase space. Then Liapunov stability techniques are applied.

Lions [Lio77] proposed asymptotic methods used to simplify situation where the size of the problem is large. This results in simpler numerical computations. The methods are used when the data has coefficients with different orders of magnitude. Three cases, including examples for each case, are given at the beginning. They are,

- Perturbations of the state equation
- Perturbations of the cost function
- degeneracy of the cost function (cheap control).

The perturbation of the domain is also considered.

Reviews of applications of distributed parameter control theory, mostly in chemical engineering, can be found in [Ray77]. References given are tabulated and classified according to the nature of the work into system identification, state estimation, real time control, and optimal design. Applications mentioned include process control studies, mechanical systems, resource recovery and the environment, physiological and sociological systems. The reason why it is necessary to resort to the theory of distributed parameter control. Also the reason why DPC is not being implemented in real time control of industrial processes is given.

Extremal eigenvalues represent boundaries of the domain of parametric stability, for example the critical buckling loads in structural design problems. A pair of variational inequalities, which are used to obtain upper and lower bounds on such values, is given in [Bar77]. q A maximum principle for the optimal control of DPS is given in [Gre73]. A set of coupled first order equations is considered. This can be reduced from a high-order partial differential equation, though with some

cost in terms of redundant solutions. Then it is proved that a minimising function is minimized by minimising a corresponding Hamiltonian. First the case where boundary conditions satisfy a matching condition is considered. Then extensions are made to these cases, namely free boundary controls, system with higher order equations, and system with more than one independent variables. Distributed controls affect the characteristics of the partial differential equations, thus leading to non-unique controls. Obtained are regions of the solution domain where the state and co-state functions are not uniquely determined.

In [FFA71], the generalized circle criterion is combined with the coerciveness theory of elleptic boundary value problems. The L_2-stability theorem for the problems is obtained.

Infinite dimensional systems are those described by differential equations in Banach space, examples of which are distributed parameter systems described by partial differential equations, integrodifferential equations and equations with delays. Unlike finite-dimensional plants, Lipschitzian condition is not generally satisfied for plants which have infinite-dimension. A control case with limitations and assumptions is obtained in [OU87]. More general cases are not possible because of both the lack of the sliding mode existence conditions, and the lack of mathematical description of sliding modes.

Polytope of polynomials

On the Pseudoboundary of Unstable Polytopes of Polynomials, by Pujara

Abstract

Pseudoboundary is defined to be the set of all polynomials in the polytope each of which has at least one zero on the imaginary axis. From this paper [Puj96]

- a section of the pseudoboundary corresponding to ω_0 is itself a polytope whose vertices lie in the exposed 2-d faces of the given polytope,

- an algorithm to generate all the vertex polynomials of any section of the pseudoboundary of a polytope.

Introduction

Polytope is stable if and only if its exposed edges are stable. Kharitonov's results is that

> If a polynomial family consists of *interval polynomials* (ie. if the family is a hyperrectangle in coefficient space with edges parallel to the coordinate axis), then the stability of just four extreme polynomials is both necessary and sufficient for the stability of the entire rectangle.

- *stability boundary problem* is still an open problem,

- this paper is related to this problem.

Problem formulation and the main results

$$f(s,q) = s^n + a_1(q)s^{n-1} + a_2(q)s^{n-2} + \cdots + a_{n-1}(q)s + a_n(q) \tag{1}$$

where $a_i(q)$, $i = 1\, to\, n$ are real affine coefficients and q has r componenets $q_1, q_2, \cdots, q_r$, and

$$q_i^- \leq q_i \leq q_i^+, \quad 1 \leq i \leq r. \tag{2}$$

Then

- for fixed q, a polynomial given by Equation 1 is a point in $\mathbb{R}^{n+1}$ with the coefficients of the polynomial as its coordinates,
- assume that these coefficients are positive,
- the polynomial 1 produces an r-dimensional polytope $\mathcal{P}$ in $\mathbb{R}^{n+1}$,
- the set of polynomials in the polytope $\mathcal{P}$, each having at least one zero on the imaginary axis, is its *pseudoboundary* β,
- β_0 is a *section* of β at ω_0 and consists of those polynomials each of which has a zero at $j\omega_0$,
- let $\mathcal{Z} \in \mathbb{C}^{n+1}$ be a set of all zeros of a polynomial 1, then $\omega_0 \in W$ if and only if $\exists g(j\omega 0) \in \mathcal{P}$ such that $g(j\omega_0) = 0$,
- this paper gives explicit description of all sections of the pseudoboundary,
- every $\beta(j\omega_0)$ is a polytope having its vertices on the exposed 2-D faces of $\mathcal{P}$,
- these vertices can be determined explicitly.

Theorem 0.0.1 *The section of the pseudoboundary β of a polytope $\mathcal{P}$ at any frequency ω_0 is a polytope.*
The vertices of this pseudoboundary lie in the exposed 2-D faces of the polytope $\mathcal{P}$.

Proof

- set of all polynomials $p(s)$ such that $p(j\omega_0) = 0$ is an $(n-2)$-dimensional affine set, $\mathcal{A}$.
- $\mathcal{A}$ is convex.
- $\mathcal{A} \cap \beta = \beta(\omega_{\prime})$ is convex.
- the vertices of $\beta(\omega_0)$ lie in the exposed faces of the given polytope.
  $\square$
  - $f(s) = (s - s_c)(s - \bar{s}_c)(s^{n-2} + d_1 s^{n-3} + \cdots + d_{n-2})$
  - L_r an $(n-2)$-dimensional affine set of all polynomials having $s_c, \bar{s}_c$ as a root
  - assume $m = \dim(\Omega) \geq 3$ $\square$ $m = 2$ is of no interest since then $s_c, \bar{s}_c$ are the only roots #

- $\dim(L_r \cap \text{aff}(\Omega)) \geq 2$
- $L_r \cap \text{aff}(\Omega)$ pierces the relative boundary of Ω
- the relative boundary comprises of some $(m-2)$-dimensional polytopes
- one of these polytopes Ω_{m-2} satisfies $s_c, \bar{s}_c \in R(\Omega_{m-2})$
- if $\dim(\Omega_{m-2}) \geq 3$, then $L_r \cap \text{aff}(\Omega_{m-1})$ intersects the relative boundary of Ω_{m-2}
- $\exists$ polytope Ω_{m-3} contained in relative boundary of Ω_{m-2} such that $s_c, \bar{s}_c \in R(\Omega_{m-2})$
- $\cdots$
- $\Omega_2 \rightarrow$ which is an *exposed face*

#

The algorithm for a 2-D Polytope

Let L be a simplex

$$L = \left\{ (l_1, l_2, \cdots, l_p)', \quad 0 \leq l_i \leq 1, \quad \sum_{i=1}^{p} l_i = 1 \right\} \tag{3}$$

where l' is the transpose of l. Suppose

$$\mathcal{R}_0(\omega_0) = \text{Re}(V_1(j\omega_0), V_2(j\omega_0), \cdots, V_p(j\omega_0))'. \tag{4}$$

For each edge E of the simplex L, find the vertices of the intersection of E and the hyperplane

$$l'\mathcal{R}_0(\omega_0) = 0, l \quad \in L. \tag{5}$$

Example, for the edge associated with the vertices $V_1(s)$ and $V_2(s)$,

$$\left.\begin{array}{lrcl} & l_1\text{Re}(V_1(j\omega_0)) + l_2 Re(V_2(j\omega_0)) = 0 & = & 0 \\ \text{and} & l_1 + l_2 & = & 1 \end{array}\right] . \tag{6}$$

$$\text{Re}(h_i(j\omega_0)) = 0, \quad 1 \leq i \leq r. \tag{7}$$

Let

$$I_0(\omega_0) = \text{Im}(h_1(j\omega_0), h_2(j\omega_0), \cdots, h_r(j\omega_0))'. \tag{8}$$

$$\text{m}' \cdot I_0(\omega_0) = 0, \quad \text{m} \in M. \tag{9}$$

Example, for an edge associated with $h_1(s)$ and $h_2(s)$

$$\left.\begin{array}{rcl} m_1\text{Im}(h_1(j\omega_0)) + m_2\text{Im}(h_2(j\omega_0)) & = & 0 \\ m_1 + m_2 & = & 1 \end{array}\right] . \tag{10}$$

$\square$ Example,

$$\begin{array}{rcccl} h_{12} & = & l_1 f_1(j\omega_0) + l_2 f_2(j\omega_0) & = & \alpha j \\ h_{34} & = & l_3 f_3(j\omega_0) + l_4 f_4(j\omega_0) & = & \beta j \end{array}$$

$$\begin{array}{rcl} m_1\text{Im}h_{12}(j\omega_0) + m_2\text{Im}h_{34}(j\omega_0) & = & 0 \\ m_1\alpha + m_2\beta & = & 1 \end{array}$$

#

$$g_i(j\omega_0) = 0, \quad 1 \le i \le k. \tag{11}$$

$$G = g_1(s), g_2(s), \cdots, g_k(s). \tag{12}$$

$\beta_F(\omega_0)$ is the convex hull of the elements of G.
Proof

- If all the polynomials in the face F has a zero at $j\omega_0$, then G consists of all the vertex polynomials of F.
  Delete from G polynomials which are not vertex-polynomials of F.
  □ $\forall l_1, l_2, l_3, l_4 \in [0, 1]$

$$\begin{aligned} l_1 \mathrm{Re} V_1 + l_2 \mathrm{Re} V_2 + l_3 \mathrm{Re} V_3 + l_4 \mathrm{Re} V_4 &= 0 \\ l_1 + l_2 + l_3 + l_4 &= 1 \end{aligned}$$

  #

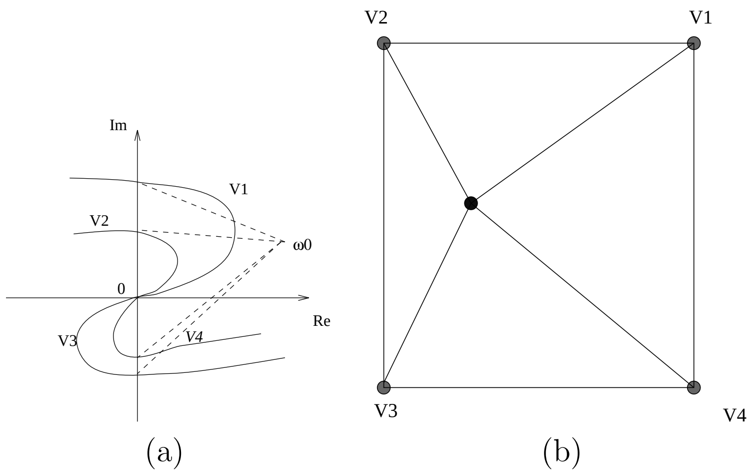


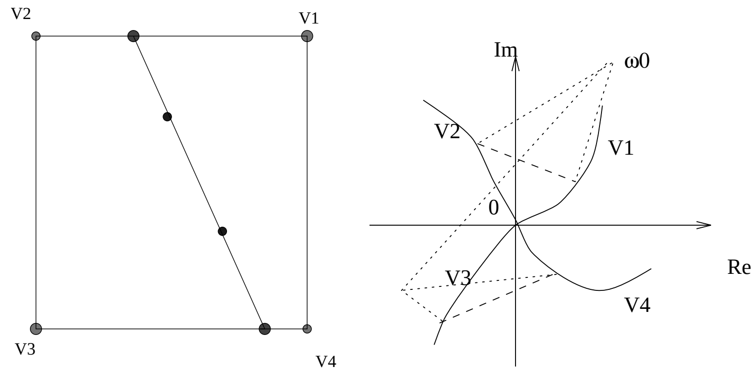
Figure 1: When every polynomials on the face F has a zero at $j\omega_0$

- If F has two polynomials vanishing at $j\omega_0$,
  the end points of the line through these two points are on the edges of F.
  Only these endpoint polynomials remain.
  □ #

- If F has only one polynomial which has a zero at $j\omega_0$, then G consists of that polynomial.
  □ $\exists$uniquel_1, l_2, l_3, l_4

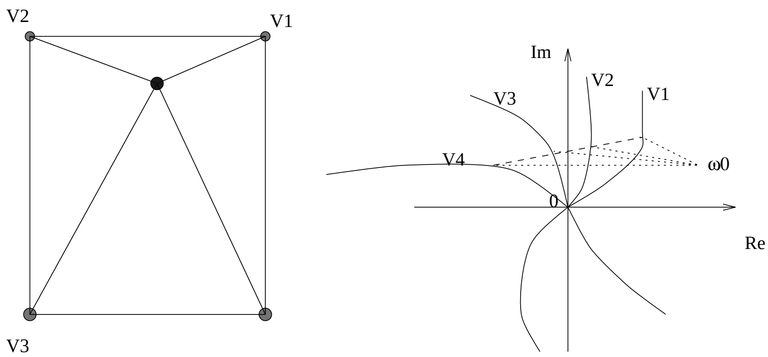
$$\begin{aligned} l_1 \mathrm{Re} V_1 + l_2 \mathrm{Re} V_2 + l_3 \mathrm{Re} V_3 + l_4 \mathrm{Re} V_4 &= 0 \\ l_1 + l_2 + l_3 + l_4 &= 1 \end{aligned}$$

  #



(a) (b)

Figure 2: When two polynomials on the face F has a zero at $j\omega_0$



(a)

Figure 3: When two polynomials on the face F has a zero at $j\omega_0$

Example: An interval polytope $\mathcal{P}$ generated by the interval polynomial

$$s^4 + [0.5, 4]s^3 + [1, 6]s^2 + [0.5, 6]s + [0.5, 3]. \tag{13}$$

By using Kharitonov's theorem

$$\begin{aligned} K^1(s) &= s^4 + 4S^3 + 6S^2 + 0.5S + 0.5 \\ K^2(s) &= s^4 + 0.5s^3 + 6s^2 + 6s + 0.5 \\ K^3(s) &= s^4 + 4s^3 + s^2 + 0.5s + 3 \\ K^4(s) &= s^4 + 0.5s^3 + s^2 + 6s + 3. \end{aligned}$$

Using Routh's test $(1) \cdot (.5) - (6) \cdot (4) \leq 0$, therefore the interval polytope $\mathcal{P}$ is not stable. The number of exposed 2-D faces of the interval polytope is $\begin{pmatrix} 4 \\ 2 \end{pmatrix}(2)(2) = 24$. $\Box$ affine $\rightarrow$ linear coefficient, constant coefficient # $\Box$ $\begin{pmatrix} 4 \\ 2 \end{pmatrix} \cdot 2 \cdot 2 = 24$#

$$\left.\begin{array}{rcl} f_1(s) & = & s^4+0.5s^3+s^2+0.5s+0.5 \\ f_2(s) & = & s^4+0.5s^3+6s^2+0.5s+0.5 \\ f_3(s) & = & s^4+0.5s^3+6s^2+6s+0.5 \\ f_4(s) & = & s^4+0.5s^3+s^2+6s+0.5 \end{array}\right] . \tag{14}$$

□

$$\begin{gathered} \mathrm{Re}(f_1(j)) = \mathrm{Re}(1-j0.5-1+j0.5+0.5) = 0.5 \\ \mathrm{Re}(f_2(j)) = \mathrm{Re}(1-j0.5-6+j0.5+0.5) = -4.5 \\ 0.5l_1 - 4.5l_2 = 0 \\ l_1 + l_2 = 1 \\ l_1 = 0.9 \quad l_2 = 0.1 \\ h_{12} = 0.9(s^4+0.5s^3+s^2+0.5s+0.5) + 0.1(s^4+0.5s^3+6s^2+0.5s+0.5) \end{gathered}$$

#

$$h_{12}(s) = s^4+0.5s^3+1.5s^2+0.5s+0.5. \tag{15}$$

□

$$\begin{gathered} \mathrm{Re}(f_3(j)) = -4.5 \\ \mathrm{Re}(f_4(j)) = 0.5 \\ -4.5l_3 + 0.5l_4 = 0 \\ vl_3 + l_4 = 1 \\ l_3 = 0.1 \quad l_4 = 0.9 \end{gathered}$$

#

$$h_{34}(s) = s^4+0.5s^3+1.5s^2+6s+0.5 \tag{16}$$

□

$$\begin{gathered} -4.5l_2 - 4.5l_3 = 0 \\ \text{but} \quad l_2 + l_3 = 1 \\ \text{no solution} \\ 0.5l_4 + 0.5l_1 = 0 \\ l_4 + l_1 = 1 \\ \text{no solutions} \end{gathered}$$

#

□ faces → vertices (V, f) → edges (h)#

$$g_1(s) = s^4+0.5s^3+1.5s^2+0.5s+0.5. \tag{17}$$

$$\left.\begin{array}{l} s^4+0.5s^3+s^2+0.5s+0.5 \\ s^4+0.5s^3+6s^2+0.5s+0.5 \\ s^4+0.5s^3+6s^2+0.5s+3 \\ s^4+0.5s^3+s^2+0.5s+3 \end{array}\right] \tag{18}$$

$$\left.\begin{array}{rcl} g_2(s) & = & s^4+0.5s^3+1.5s^2+0.5s+0.5 \\ g_3(s) & = & s^4+0.5s^3+4s^2+0.5s+3 \end{array}\right] \tag{19}$$

□

	f								h							
No.	s^4	s^3	s^2	s^1	s^0	Re	l_1	l_2	s^4	s^3	s_2	s_1	s_0	Im	m_1	m_2
1	1	0.5	1	0.5	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	0.5	0.5	0.5	0.9	0.1	1	4	1.5	0.5	0.5	−3.5	N/A	N/A
	1	4	6	0.5	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	0.5	0.5	−4.5	0.1	0.9	1	0.5	1.5	0.5	0.5	0	N/A	N/A
2	1	0.5	1	0.5	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	0.5	3	3	0.4	0.6	1	4	4	0.5	3	−3.5	N/A	N/A
	1	4	6	0.5	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	0.5	3	−2	0.6	0.4	1	0.5	4	0.5	3	0	N/A	N/A
3	1	0.5	1	6	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	6	0.5	0.5	0.9	0.1	1	4	1.5	6	0.5	2	N/A	N/A
	1	4	6	6	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	6	0.5	−4.5	0.1	0.9	1	0.5	1.5	6	0.5	5.5	N/A	N/A
4	1	0.5	1	6	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	6	3	3	0.4	0.6	1	4	4	6	3	2	N/A	N/A
	1	4	6	6	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	6	3	−2	0.6	0.4	1	0.5	4	6	3	5.5	N/A	N/A
5	1	0.5	1	0.5	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	0.5	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	6	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	1	6	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
6	1	0.5	1	0.5	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	0.5	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	6	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	1	6	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
7	1	0.5	6	0.5	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	0.5	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	6	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	6	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
8	1	0.5	6	0.5	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	0.5	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	6	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	6	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
9	1	0.5	1	0.5	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	0.5	0.5	0.5	1.2	−0.2	1	4	1	0.5	0	−3.5	N/A	N/A
	1	4	1	0.5	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	1	0.5	3	3	−0.2	1.2	1	0.5	1	0.5	0	0	N/A	N/A
10	1	0.5	1	6	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	6	0.5	0.5	1.2	−0.2	1	4	1	6	0	2	N/A	N/A
	1	4	1	6	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	1	6	3	3	−0.2	1.2	1	0.5	1	6	0	5.5	N/A	N/A
11	1	0.5	6	0.5	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	0.5	0.5	−4.5	−0.8	1.8	1	4	6	0.5	5	−3.5	N/A	N/A
	1	4	6	0.5	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	0.5	3	−2	1.8	−0.8	1	0.5	6	0.5	5	0	N/A	N/A
12	1	0.5	6	6	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	6	0.5	−4.5	−0.8	1.8	1	4	6	6	5	2	N/A	N/A
	1	4	6	6	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	6	3	−2	1.8	−0.8	1	0.5	6	6	5	5.5	N/A	N/A

	g				
$No.$	s^4	s^3	s^2	s^1	s_0
1	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
2	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
3	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
4	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
5	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
6	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
7	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
8	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
9	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
10	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
11	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
12	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A

	f								h							
No.	s^4	s^3	s^2	s^1	s^0	Re	l_1	l_2	s^4	s^3	s_2	s_1	s_0	Im	m_1	m_2
13	1	0.5	1	0.5	0.5	0.5	0.9	0.1	1	0.5	1.5	0.5	0.5	0	1	0
	1	0.5	6	0.5	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	6	0.5	−4.5	0.1	0.9	1	0.5	1.5	6	0.5	5.5	N/A	N/A
	1	0.5	1	6	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
14	1	0.5	1	0.5	3	3	0.4	0.6	1	0.5	4	0.5	3	0	1	0
	1	0.5	6	0.5	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	6	3	−2	0.6	0.4	1	0.5	4	6	3	5.5	N/A	N/A
	1	0.5	1	6	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
15	1	4	1	0.5	0.5	0.5	0.9	0.1	1	4	1.5	0.5	0.5	−3.5	0.36	0.64
	1	4	6	0.5	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	6	0.5	−4.5	0.1	0.9	1	4	1.5	6	0.5	2	N/A	N/A
	1	4	1	6	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
16	1	4	1	0.5	3	3	0.4	0.6	1	4	4	0.5	3	−3.5	0.36	0.64
	1	4	6	0.5	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	6	3	−2	0.6	0.4	1	4	4	6	3	2	N/A	N/A
	1	4	1	6	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
17	1	0.5	1	0.5	0.5	0.5	0.9	0.1	1	0.5	1.5	0.5	0.5	0	N/A	N/A
	1	0.5	6	0.5	0.5	−4.5	−0.8	1.8	1	0.5	6	0.5	5	0	N/A	N/A
	1	0.5	6	0.5	3	−2	0.6	0.4	1	0.5	4	0.5	3	0	N/A	N/A
	1	0.5	1	0.5	3	3	−0.2	1.2	1	0.5	1	0.5	0	0	N/A	N/A
18	1	0.5	1	6	0.5	0.5	0.9	0.1	1	0.5	1.5	6	0.5	5.5	N/A	N/A
	1	0.5	6	6	0.5	−4.5	−0.8	1.8	1	0.5	6	6	5	5.5	N/A	N/A
	1	0.5	6	6	3	−2	0.6	0.4	1	0.5	4	6	3	5.5	N/A	N/A
	1	0.5	1	6	3	3	−0.2	1.2	1	0.5	1	6	0	5.5	N/A	N/A
19	1	4	1	0.5	0.5	0.5	0.9	0.1	1	4	1.5	0.5	0.5	−3.5	N/A	N/A
	1	4	6	0.5	0.5	−4.5	−0.8	1.8	1	4	6	0.5	5	−3.5	N/A	N/A
	1	4	6	0.5	3	−2	0.6	0.4	1	4	4	0.5	3	−3.5	N/A	N/A
	1	4	1	0.5	3	3	−0.2	1.2	1	4	1	0.5	0	−3.5	N/A	N/A
20	1	4	1	6	0.5	0.5	0.9	0.1	1	4	1.5	6	0.5	2	N/A	N/A
	1	4	6	6	0.5	−4.5	−0.8	1.8	1	4	6	6	5	2	N/A	N/A
	1	4	6	6	3	−2	0.6	0.4	1	4	4	6	3	2	N/A	N/A
	1	4	1	6	3	3	−0.2	1.2	1	4	1	6	0	2	N/A	N/A
21	1	0.5	1	0.5	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	1	6	0.5	0.5	1.2	−0.2	1	0.5	1	6	0	5.5	N/A	N/A
	1	0.5	1	6	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	1	0.5	3	3	−0.2	1.2	1	0.5	1	0.5	0	0	N/A	N/A
22	1	0.5	6	0.5	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	6	0.5	−4.5	−0.8	1.8	1	0.5	6	6	5	5.5	N/A	N/A
	1	0.5	6	6	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	0.5	6	0.5	3	−2	1.8	−0.8	1	0.5	6	0.5	5	0	N/A	N/A
23	1	4	1	0.5	0.5	0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	6	0.5	0.5	1.2	−0.2	1	4	1	6	0	2	N/A	N/A
	1	4	1	6	3	3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	1	0.5	3	3	−0.2	1.2	1	4	1	0.5	0	−3.5	N/A	N/A
24	1	4	6	0.5	0.5	−4.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	6	0.5	−4.5	−0.8	1.8	1	4	6	6	5	2	N/A	N/A
	1	4	6	6	3	−2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	1	4	6	0.5	3	−2	1.8	−0.8	1	4	6	0.5	5	−3.5	N/A	N/A

	g				
$No.$	s^4	s^3	s^2	s^1	s_0
13	1	0.5	1.5	0.5	0.5
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
14	1	0.5	4	0.5	3
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
15	1	4	1.5	4	0.5
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
16	1	4	4	4	3
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
17	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
18	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	1	0.5	N/A	0	N/A
	1	0.5	N/A	0	N/A
19	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
20	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	1	4	N/A	4	N/A
	1	4	N/A	4	N/A
21	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
22	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
23	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
24	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A
	N/A	N/A	N/A	N/A	N/A

#

$$\left.\begin{array}{rcl} g_4(s) & = & s^4 + 4s^3 + 1.5s^2 + 4s + 0.5 \\ g_5(s) & = & s^4 + 4s^3 + 4s^2 + 4s + 3 \end{array}\right]. \tag{20}$$

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